HBT

generally the correlationfunction is defined as

$$C_2 \equiv \frac{P(p_1 p_2)}{P(p_1) P(p_2)}$$

it contains information of the spatial evolution of the particle source

$$C_2 \approx \frac{\left| \int S(x,k) \exp(iqx) d^4 x \right|^2}{\left| \int S(x,k) d^4 x \right|^2}$$

choose a parameterization of the correlationfunction assuming a set of conditions and fit it to the data

$$C_2 \approx 1 + \lambda \exp\left(\sum q_{ij} R_{ij} \right)$$

the fit parameters (= HBT radii) then have a meaning within the assumed picture

Most simple case

assume we have :

- a very Gaussian source : $\rho(r) = \frac{1}{\pi^2 R_{inv}^{\ 4}} \exp\left(\frac{-r^2}{R_{inv}^{\ 2}}\right)$
- no final state interactions (e.g. Coulomb ...)
- complete chaotic (= independent) emission
- complete static source (e.g. no flow, ...)
- pure pion source (no resonances, ...)
- no experimental influences
- ...

then HBT enables us to measure R :

$$C_2 = 1 + \exp\left(-q_{inv}^2 R_{inv}^{\ 2}\right)$$

Yano Koonin Podgoretskii pararameterization

assume a source which is

- static
- azimuthal symmetric
- gaussian shaped

the adequate parameterization would then be :

$$C_2(Q) = C_2(q_{perp}, q_{II}, q_0) = 1 + \exp(-q_{perp}^2 R_{perp}^2 - q_{II}^2 R_{II}^2 - q_0^2 R_0^2)$$

with

$$q_x = p_{x,1} - p_{x,2} \qquad q_{perp} = \sqrt{q_x^2 + q_y^2}$$

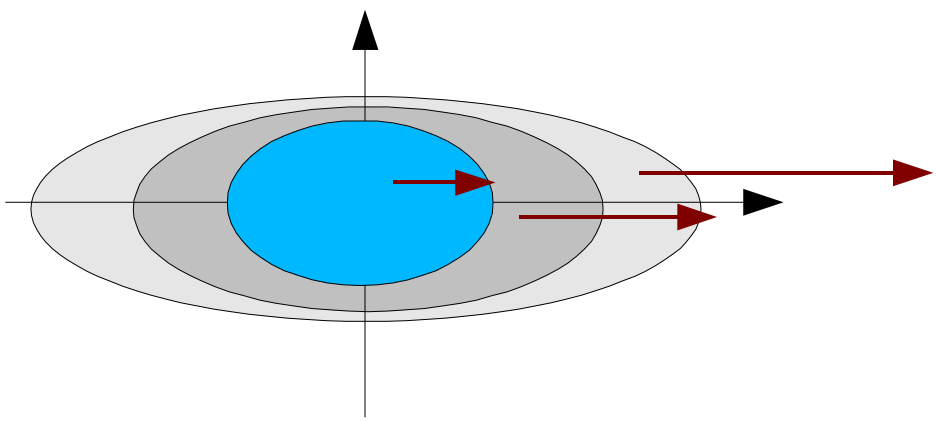
$$q_0 = E_1 - E_2 \qquad q_{II} = p_{z,1} - p_{z,2}$$

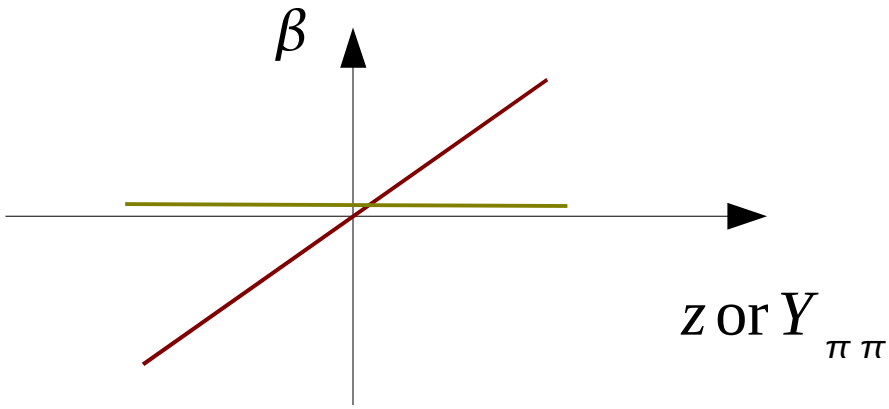
but if there is a strong longitudinal expansion it is visible as a relativistic boost along the beam:

$$C_2 = 1 + \exp(-q_{perp}^2 R_{perp}^2 - \gamma^2 (q_{long} - \beta q_0)^2 R_{long}^2 - \gamma^2 (q_0 - \beta q_{long})^2 R_0^2)$$

Yano Koonin Podgoretskii parameterization

$$C_2 = 1 + \exp\left(-q_{perp}^2 R_{perp}^2 - \gamma^2 (q_{long} - \beta q_0)^2 R_{long}^2 - \gamma^2 (q_0 - \beta q_{long})^2 R_0^2\right)$$





Yano Koonin Podgoretskii parameterization

But, what do the radii mean ?

assume the phase space density to be :

$$S(x,k) = S(\bar{x},k)\exp(-\tilde{x}^m B_{mn}\tilde{x}^n)$$

with

$\bar{x}$: point of maximum emission

$\tilde{x}$: distance to point of maximum emission $\bar{x}$

and choose the correlationfunction:

$$C_2 = 1 + \exp(-q^m q^n \langle \tilde{x}_m \tilde{x}_n \rangle) = 1 + \exp(-q^m q^n B^{-1})$$

$$R_{perp} = \langle \tilde{y}^2 \rangle$$

$$R_{II}^2 = \langle (\tilde{z} - \frac{\beta_{II}}{\beta_{perp}}\tilde{x})^2 \rangle - \frac{\beta_{II}}{\beta_{perp}}\langle \tilde{y}^2 \rangle \approx \langle \tilde{z}^2 \rangle$$

$$R_0^2 = \langle (\tilde{t} - \frac{1}{\beta_{perp}}\tilde{x})^2 \rangle - \frac{1}{\beta_{perp}}\langle \tilde{y}^2 \rangle \approx \langle \tilde{t}^2 \rangle$$

Bertsch Pratt parameterization

$$C_2(Q) = C_2(q_{side}, q_{out}, q_{long}) = 1 + \exp(-q_{side}^2 R_{side}^2 - q_{out}^2 R_{out}^2 - q_{long}^2 R_{long}^2)$$

the approximations are then

$$R_{side} = \langle \tilde{y}^2 \rangle$$

$$R_{out}^2 = \langle (\tilde{x} - \beta_{perp} \tilde{t})^2 \rangle$$

$$R_{long}^2 = \langle (\tilde{z} - \beta_{II} \tilde{t})^2 \rangle$$

$\tilde{x}, \tilde{y}, \tilde{z}, \tilde{t}$: distance to point of maximum emission

β_{perp} : transverse expansion

β_{II} : longitudinal expansion

and from that the lifetime might be measured

$$R_{out}^2 - R_{side}^2 \approx \beta_{perp}^2 \langle \tilde{t}^2 \rangle$$

BP vs YKP

the radii from different parameterizations are related in the following way :

$$R_{side}^2 = R_{perp}^2$$

$$R_{out}^2 - R_{side}^2 = \beta_{perp}^2 \gamma^2 (R_0^2 + \beta_{YKP}^2 R_{II}^2)$$

$$R_{long}^2 = (1 - \beta_{II}^2) R_{II}^2 + \gamma^2 (\beta_{II} - \beta_{YKP})^2 (R_0^2 + R_{II}^2)$$

in LCMS $\beta_{II} = 0$:

$$R_{out}^2 - R_{side}^2 = \beta_{perp}^2 \gamma^2 (R_0^2 + \beta_{YKP}^2 R_{II}^2)$$

$$R_{long}^2 = \gamma^2 (R_{II}^2 + \beta_{YKP}^2 R_{II}^2)$$

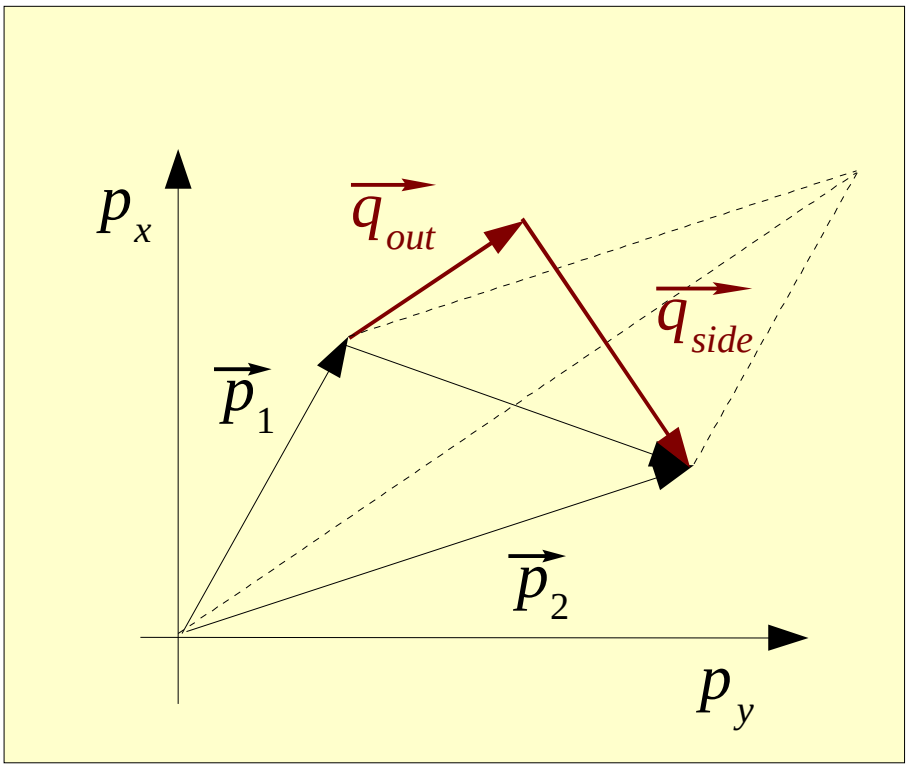
and in a boostivariant source $\beta_{YKP} = 0$:

$$R_{out}^2 - R_{side}^2 = \beta_{perp}^2 R_0^2$$

$$R_{long}^2 = R_{II}^2$$

Bertsch Pratt parameterization

$$C_2(Q) = C_2(q_{side}, q_{out}, q_{long}) = 1 + \exp(-q_{side}^2 R_{side}^2 - q_{out}^2 R_{out}^2 - q_{long}^2 R_{long}^2)$$



q_{side}, q_{out} : see figure

$$q_{long} = p_{z,1} - p_{z,2}$$

Summary

- different parameterizations of the correlationfunction address different aspects of the emission source
- fit parameters must be interpreted within the underlying picture, they do not have necessarily a simple intuitive meaning
- if the radii obtained from YKP and from BP are consistent, the interpretation of their meaning may become more meaningfull.....
- next time you will hopefully see more data :)